# *SYNOPSIS*

The thesis entitled“Generalized Burgers’ equation and its application in various industrial problems” is an attempt to study the generalized Burgers’ equation and its applications to real life situations & various industrial problems, to formulate the non – linear problem using generalized Burgers’ equation and compare the model with the existing model and to analyze the flexibility of generalized Burgers’ equation.

It consists of five chapters. Chapter 1 is an introduction of Burgers’ equation and generalized Burgers’ equation. Chapter 2 contains analysis of Homotopy Perturbation method. Further, chapter 3 contains analytical solution of Burgers’ equation, as well as numerical solution of Burgers’ equation and generalized Burgers’ equation by Homotopy Perturbation method, graphs and finally interpretation of the problem from the graphs obtained. Chapter 4 comprises of problems of homogeneous and non homogeneous equations pertaining to application in industrial problems. Last chapter i.e. chapter 5 is the conclusion of the research work.

Broadly speaking, Chapter 1 provides the fundamentals of Fluid mechanics, Partial differential equation, Burgers’ equation and generalized Burgers’ equation. Burgers' equation is a fundamental partial differential equation from fluid mechanics. It occurs in various areas of applied mathematics, such as modeling of gas dynamics, traffic flow, liquid flow, non linear electrical circuit element and attenuation of plane wave in thermoviscous medium.

For a given velocity u and viscosity coefficient *v*, Burgers' equation is:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = v \frac{\partial^2 u}{\partial x^2}$$

It is the simplest model of diffusive waves which under certain simplifying assumptions covers turbulence, sound waves in viscous media, waves in fluid-filled visco - elastic tubes and magneto-hydrodynamic waves in media having finite electrical conductivity.

Burgers' equation was proposed as a mathematical model of turbulence. (Bateman, 1915)

The Burgers' equation is an idealized equation which combines a simple non-linearity viscous term. But in actual physical situations there are other complicating physical factors which alter Burgers' equations. These contributing terms may be lower order source or sink terms or geometrical expansion terms like spherical or cylindrical waves which arise due to point explosion such as blast wave or disturbance produced along a line called cylindrical wave.

The other very important topic of our work is generalized Burgers' equation.
generalized Burgers' equation is:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \lambda u = v \frac{\partial^2 u}{\partial x^2}$$

A partial differential equation is always connected with a particular type of associated conditions. It is well-known fact that non linear partial differential equations cannot be integrated analytically. Inthese cases it is necessary to apply some method of approximation such as Homotopy Perturbation Method.

It is well known that the exact solution of Burgers' equation can only be computed for restricted values of ε which represent the kinematics viscosity of the fluid motion. Because of this fact, various numerical methods were employed to obtain the solution of Burgers' equation with small ε values.

In Chapter 2, Homotopy Perturbation method is described. In recent years, the Homotopy Perturbation method (HPM), first proposed by Ji Huan He (1999), was developed and improved by Ji Huan He (2006). It has been successfully applied to solve many types of linearand nonlinear functional equations. This method, which is a combination of Homotopy in topology and classic perturbation techniques, provides us with aconvenient way to obtain analytic or approximate solutions to a wide varietyof problems arising in different fields.

The combination of the Perturbation method and the Homotopy method is called the Homotopy Perturbation Method, which eliminates the drawbacks of the traditional Perturbation methods while keeping all its advantages.This method is useful to solve linear as well as non linear problems. The method can be briefly described as follows.

Given differential equation,

$$A(u) - f(x) = 0$$

Usually Homotopy Perturbation method is applied where A is a non linear differential operator which can be divided into L and N, where L is linear part and N is non linear part. In case if the given equation is linear then L is the part which is easy to solve while N is the one which is difficult to solve. Hence, Homotopy Perturbation method is a straightforward and convenient method for both linearand nonlinear equations.

Chapter 3 deals with the problems of the one-dimensional non linear partial differential equation. It includes analytical solution of Burgers' equation, numerical solution of Burgers' equation and generalized Burgers' equation by Homotopy Perturbation method. It also contains comparison of exact solution of Burgers' equation, and numerical solution of Burgers' equation and generalized Burgers' equation through table values as well as graphs.

In problem 1, we have taken the initial condition as $u(x, 0) = \sin \pi x$ and boundary conditions as $u(0,t) = u(1,t) = 0$. For this condition, exact solution of Burgers' equation and numerical solution of Burgers' equation and generalized Burgers' equation are found alongwith their graphs.

In problem 2, another initial condition $u(x, 0) = \cos(\pi x/2)$ and boundary conditions

$u(-1, t) = u(1, t) = 0$ are taken and similar procedure as above is repeated.

We have also found the general form of the u(x, t), such that by substituting any initial condition and boundary conditions, we can obtain the solution of Burgers' equation and generalized Burgers' equation by Homotopy Perturbation method.

Chapter 4 discusses the examples of homogeneous and non homogeneous equations which are presented to show the applications of generalized Burgers' equation in industries.

In the same, we discuss the modeling of industrial problems which have been transformed to non homogeneous Burgers' equation with initial and/or boundary conditions and we solve them by Homotopy Perturbation method. Also we compare the solution of Burgers' equation with the generalized Burgers' equation and the exact solution which gives the applicability of generalized Burger's equation in industrial problems.

In the last chapter, there is conclusion of the research work carried out as well as the further scope of work.The results show that Homotopy Perturbation method is a powerful mathematical tool for solving Burgers' equation and generalized Burgers' equation, and very effective, convenient and quite accurate to systems of partial differential equations. They provide more realistic series solutions that converge very rapidly in real physical problems.

Finally, in the appendix we depict all the programs of Matlab, through which we hadobtained the analytical solution of Burgers' equation and numerical solution of Burgers' equation and generalized Burgers' equation by Homotopy Perturbation method.

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